Multiple Landau level filling for a large magnetic field limit of 2D fermions

Denis Périce
denis.perice@ens-lyon.fr

PHD Advisor: Nicolas Rougerie
nicolas.rougerie@ens-lyon.fr

UMPA ENS DE LYON

cnrs

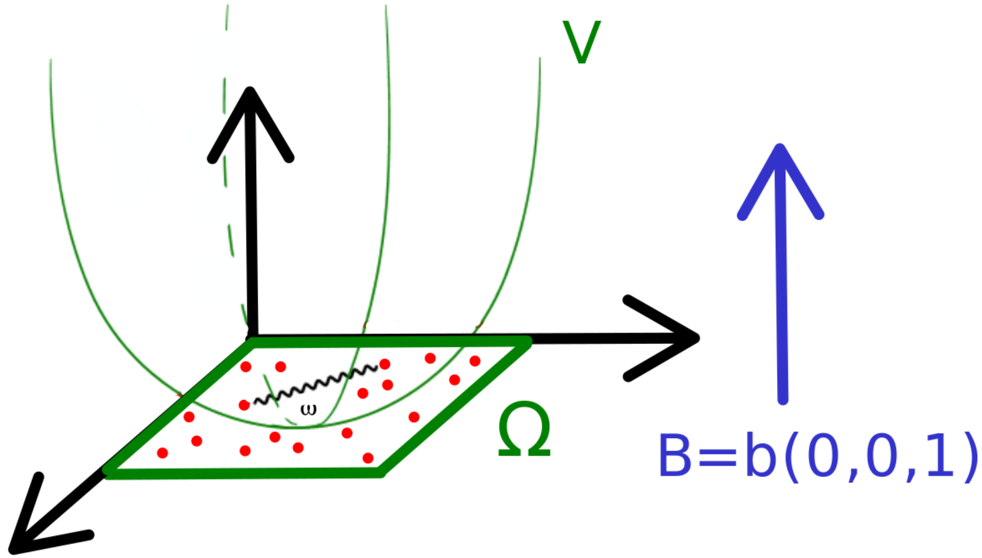
erc

Quantissima - Venice 2022

I Model

We consider N particles:

- spinless fermions
- 2D compact domain $\Omega := [0, L]^2$
- uniform transverse magnetic field B
- magnetic periodic boundary conditions



Mean field Hamiltonian

$$\mathscr{H}_N := \sum_{j=1}^{N} \left(\left(-i\hbar\nabla_j - bA(x_j)\right)^2 + V(x_j) \right) + \frac{1}{N} \sum_{1 \leqslant i < j \leqslant N} w(x_i - x_j) \tag{1}$$

Acting on

$$L^2_-\left(\Omega^N\right) := \bigwedge^N L^2(\Omega) \tag{2}$$

Coulomb gauge: $\exists \phi \in C^\infty(\Omega, \mathbb{R})$ such that

$$A = \nabla^\perp \phi \text{ and } \nabla \wedge A = (0,0,1) \tag{3}$$

We study the ground state and ground energy.

Magnetic periodic boundary conditions

Translation operator $T_y\psi(x) := \psi(x-y)$, problem:

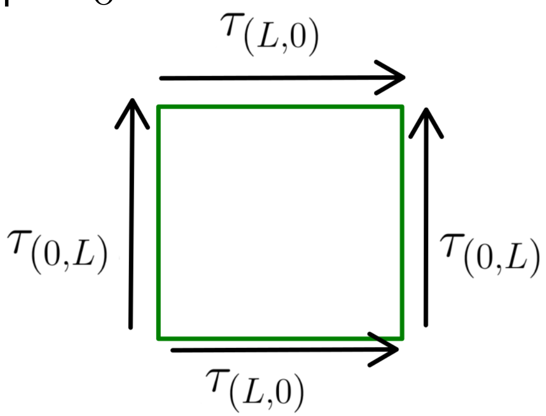
$$[T_y, i\hbar\nabla + bA] = b[T_y, A] \neq 0$$

$\nabla \wedge A = Cst \implies T_yA - A =: l_b^2\nabla\varphi_y$, define

$$\tau_y := e^{i\varphi_y}T_y \implies [\tau_y, i\hbar\nabla + bA] = 0 \tag{4}$$

Magnetic length: $l_b := \sqrt{\frac{\hbar}{b}}$

Flux quantization: $[\tau_{(L,0)}, \tau_{(0,L)}] = 0 \iff \frac{L^2}{l_b^2} \in 2\pi\mathbb{Z}$



Landau level diagonalization of the magnetic Laplacian:

$$\mathscr{L} := (i\hbar\nabla + bA)^2 = \sum_{n\in\mathbb{N}} E_n\Pi_n \text{ with } E_n := 2\hbar b\left(n + \frac{1}{2}\right) \tag{5}$$

on

$$\text{Dom}(\mathscr{L}) := \left\{\psi \in H^2(\Omega) | \forall t \in [0,L], \begin{array}{l} \psi(L,t) = e^{i\varphi_{(L,0)}(L,t)}\psi(0,t) \\ \psi(t,L) = e^{i\varphi_{(0,L)}(t,L)}\psi(t,0) \end{array}\right\} \tag{6}$$

Moreover $\text{Rank}(\Pi_n) = \frac{L^2}{2\pi l_b^2} =: d$

Characteristics lengths:

- $L/\sqrt{N}$: mean distance between particles
- l_b: minimal distance between particles

[**Lieb, Solovej, Yngvason (1995)**]: coulomb interaction

- $L^2/l_b^2 N \to +\infty$: all particles lowest Landau level
- $L^2/l_b^2 N \to 0$: all Landau level filled

[**Fournais, Lewin, Solojev (2015)**], [**Fournais, Madsen (2019)**]: general V, w

Scaling

Semi-classical limit:

$$\hbar := N^{-\delta} \text{ with } \frac{1}{4} < \delta < \frac{1}{2} \tag{7}$$

Let $q \in \mathbb{N}$, $r \in [0,1)$, fix b such that

$$\frac{N}{d} = \frac{2\pi l_b^2 N}{L^2} \underset{N\to\infty}{\longrightarrow} q + r \implies \mathcal{O}(\hbar b) = \hbar^2 N = N^{1-2\delta} \gg 1 \tag{8}$$

II Limit model

Semi-classical functional

To $m : \mathbb{N} \times \Omega \to \mathbb{R}_+$, such that

$$0 \leqslant m \leqslant \frac{1}{(q+r)L^2} \text{ and } \sum_{n \in \mathbb{N}} \int_\Omega m(n,x)dx = 1 \tag{9}$$

associate the semi-classical energy

$$\mathcal{E}_{sc,N}[m] \coloneqq \sum_{n \in \mathbb{N}} E_n \int_\Omega m(n,x)dx + \sum_{n \in \mathbb{N}} \int_\Omega V(x)m(n,x)dx \tag{10}$$

$$+ \frac{1}{2} \sum_{n,\tilde{n} \in \mathbb{N}} \iint_{\Omega^2} w(x-y)m(n,x)m(\tilde{n},y)dxdy \tag{11}$$

Model for the partially filled Landau level

Electrostatic functional:

$$\mathcal{E}_{qLL}[\rho] \coloneqq \int_{\Omega} V(x)\rho(x) + \frac{1}{2}\iint_{\Omega^2} w(x-y)\rho(x)\rho(y)dxdy \tag{12}$$

with domain

$$\mathcal{D}_{qLL} \coloneqq \left\{ \rho \in L^1(\Omega) \text{ such that } \int_{\Omega} \rho = \frac{r}{q+r} \text{ and } 0 \leqslant \rho \leqslant \frac{1}{(q+r)L^2} \right\} \tag{13}$$

Let $\rho \in \mathcal{D}_{qLL}$, define

$$m_\rho(n,x) \coloneqq \frac{1}{L^2(q+r)}\mathbb{1}_{n<q} + \rho(x)\mathbb{1}_{n=q} \tag{14}$$

Then,

$$\mathcal{E}_{sc,N}\left[m_\rho\right] = \hbar b C_1 + C_2 + \mathcal{E}_{qLL}\left[\rho\right]$$

III Results

Theorem 1: *Convergence of the ground state energy. D.P N.Rougerie (2022)*

If $V, w \in L^2$,

$$\inf_{\psi_N \in \mathrm{Dom}(\mathcal{H}_N), \|\psi_N\|=1} \frac{\langle \Psi_N | \mathscr{H}_N \Psi_N \rangle}{N} \underset{N\to\infty}{=} \inf_{\rho \in \mathcal{D}_{qLL}} \mathcal{E}_{sc,N}\left[m_\rho\right] + o(1)$$

Let Γ_N be a fermionic N-body density matrix, $k \in \mathbb{Z}$, define

$$\gamma_N^{(k)}(X_k, Y_k) \coloneqq \int_{\Omega^{N-k}} \Gamma_N(X_k, Z_{N-k}; Y_k, Z_{N-k}) dZ_{N-k} \text{ and } \quad \rho_N^{(k)}(X_k) \coloneqq \gamma_N^{(k)}(X_k, X_k) \tag{15}$$

Theorem 2: *Convergence of the reduced densities. D.P N.Rougerie (2022)*

If $V, w \in L^2$, $\exists \mu \in \mathcal{P}\left(\mathcal{D}_{qLL}\right)$ only charging minimizers of $\mathcal{E}_{qLL}$ such that $\forall k \in \mathbb{N}^*$ in the sense of Radon measures,

$$\rho_N^{(k)} \underset{N\to\infty}{\overset{*}{\rightharpoonup}} \int_{\mathcal{D}_{qLL}} \left(\frac{q}{L^2(q+r)} + \rho\right)^{\otimes k} d\mu(\rho)$$

IV Sketch of the proof

Semi classical approximation

Define
$$\Pi_{n,x} \coloneqq g_\lambda(\bullet - x)\Pi_n g_\lambda(\bullet - x) \tag{16}$$
$$m_N^{(k)}(n_1, x_1; \ldots; n_k, x_k) \coloneqq \mathrm{Tr}\left[\gamma_N^{(k)} \bigotimes_{i=1}^{k} \Pi_{n_i,x_i}\right] \tag{17}$$
$$\mathcal{E}_{sc,N}\left[m_N\right] \coloneqq \sum_{n\in\mathbb{N}} E_n \int_\Omega m(n,x)dx + \sum_{n\in\mathbb{N}} \int_\Omega V(x)m(n,x)dx \tag{18}$$
$$+\frac{1}{2}\sum_{n,\tilde{n}\in\mathbb{N}} \iint_{\Omega^2} w(x-y)m^{(2)}(n,x;\tilde{n},y)dxdy \tag{19}$$

$$\frac{\mathrm{Tr}\left[\mathscr{H}_N\Gamma_N\right]}{N} = \mathcal{E}_{sc,N}\left[m_N\right] + o(1) \tag{20}$$

Mean field limit

- Upper bound: Lieb variational principle
- Lower bound: De Finetti theorem

V Perspectives

- Obtain similar results for relativistic fermions

$$\mathscr{D} := -\begin{pmatrix} \sigma_x \\ \sigma_y \end{pmatrix} \cdot (i\hbar\nabla + bA) \tag{21}$$

- Dynamics: convergence in the mean field limit of the N body Schrodinger equation to Hartree-Fock and convergence in the semi classical to the vorticity equation

$$\partial_t \rho(t,R) = -\nabla_R^{\perp}(V(R) + w * \rho(t,R)) \cdot \nabla_R \rho(t,R) \tag{22}$$

Thanks for your attention :)

Bibliography

[1] **Y.Almog -** *Abrikosov Lattices in Finite Domains*
2005, Commun.Math.Phys., DOI: 10.1007/s00220-005-1463-x

[2] **S.Fournais, M.Lewin, J-P.Solovej -** *The semi-classical limit of large fermionic systems*
2015, arXiv:1510.01124

[3] **S.Fournais and P.Madsen -** *Semi-classical limit of confined fermionic systems in homogeneous magnetic fields*
2019, arXiv:1907.00629

[4] **Jainendra K.Jain -** *Composite fermions*
2007, Cambridge University Press

[5] **E.H.Lieb -** *Variational Principle for Many-Fermion Systems*
1981, Phys.Rev.Volume 46, Number 7

[6] **E.H.Lieb and R.Seiringer -** *The stability of matter in quantum mechanics*
2008

[7] **E.H.Lieb J-P.Solovej -** *Quantum Dots*
1994, arXiv:cond-mat/9404099v1

[8] **E.H.Lieb, J-P.Solovej, and J.Yngvason -** *Asymptotics of Heavy Atoms in High Magnetic Fields: I.Lowest Landau Band Region*
1994, Commun.Pure Appl.Math.47, 513-591

[9] **E.H.Lieb, J-P.Solovej, and J.Yngvason -** *Asymptotics of Heavy Atoms in High Magnetic Fields: II.Semi-classical Regions,*
1994, Commun.Math.Phys.161, 77-124

[10] **E.H.Lieb, J-P.Solovej, and J.Yngvason -** *Ground states of large quantum dots in magnetic fields*
1995, Phys.Rev.B, 51:10646–10665

[11] **N.Rougerie -** *Théorèmes de De Finetti, limites de champ moyen et condensation de Bose-Einstein*
2014, hal-01060125v4

[12] **N.Rougerie and J.Yngvason -** *Holomorphic quantum hall states in higher landau levels*
arXiv:1912.10904, 2019